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(54) Screening system and method for color reproduction in offset printing

(57) A screening system and method are disclosed for reproduction of images in printing. The screening angles that are used are close, but not identical, to conventional screening angles. The reproduction is nevertheless Moiré free by the fact that the deviations in angles from the conventional system are exactly offset by the deviations in line rulings. The screening system is particularly advantageous when used for combinations of screens with rational tangent angles. The Moiré free combination of rational tangent screens can be rotated by a constant angle with the amount of rotation controlled in small increments.

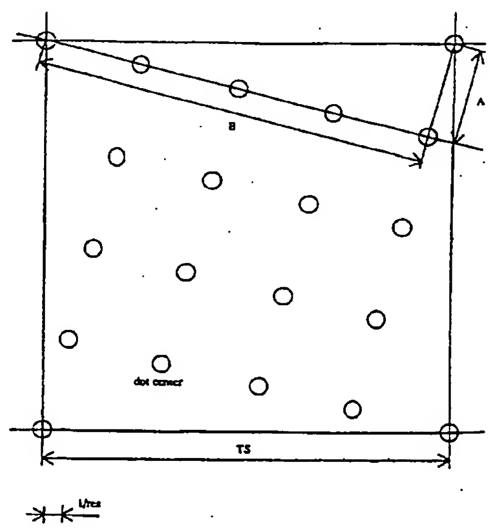

Figure 12: Geometry of a cell with preangled dots.

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Description

FIELD OF THE INVENTION

The present invention relates to screening systems in general and, more particularly, to a second order Moire free screening systems and method.

BACKGROUND OF THE INVENTION

Offset printing is a binary process the press can print INK or NO INK an a particular location of the paper. Unlike in the photographic or other "contone" reproduction processes, no controlled modulation of the ink density is possible to obtain the various shades of tone and color.

To simulate the effect of different densities necessary for the reproduction of pictures, patterns of dots are used of which the SIZE is modulated. 0% dot size corresponds to no ink, while 100% dot size corresponds to a solid ink area. The process of simulating densities by modulating dot sizes is called "halftoning".

The patterns of dots are defined by an angle (measured along the direction of the shortest line that connects two dot centers), a screen ruling (defined by the number of dot centers per measurement unit, measured in the direction of the screen angle), and the shape of the dots as they grow from 0% to 100% (usually controlled by a "spot function").

In conventional color printing, four inks are used : cyan, magenta, yellow an black. Every printable color is simulated by overprinting a particular combination of dot percentages of these four inks.

The angles and rulings of the dot patterns of these four inks are chosen with the following two considerations in mind :

1. In order to reduce the sensitivity of the color reproduction to registration errors, the relative position of the dots of the four inks has to be (pseudo) randomized.
2. It is known that geometrical patterns of dots interact with each other, and can give rise to new patterns that are referred to as "Moiré". The dot patterns in color printing should be chosen not to give rise to disturbing Moiré patterns.

In conventional technology, both of these two requirements are achieved by using dot patterns for the cyan, magenta and black inks that have exactly the same ruling, and by using angles that are exactly 30 and 60 degrees offset with respect to each other.

The yellow ink is usually printed with a screen using the same ruling as the others, and an angle that is 15 degrees offset with respect to one of the other inks. A combination of angles that works well is :

cyan	75.0 degrees
black	45.0 degrees
magenta	15.0 degrees
yellow	0.0 degrees

It can be shown that if exactly these angles are used, the same relative position of the cyan, magenta and black dots is never repeated, which meets the first requirement, while the "Moiré" is a "micro Moiré" (often referred to as the "rosette"), which, when screen rulings are used of approximately 150 l/i, is small enough not to be disturbing. See for example EP-A-0 369 243.

In general, digital screening systems or methods have been developed in the past to generate halftone dot patterns on digital film recorders. A typical digital film recorder consists of a scanning laser beam exposing a photosensitive material at high resolution. Typically the "grid" that defines the resolution at which the laser beam can be modulated ON or OFF has a pitch in the range of 1/2400 of an inch. The digital screening algorithms have to turn ON or OFF the "micro dots" on the recorder grid so that they form clusters of microdots that make up the halftone dots, the size of which corresponds to the desired local density level.

Most digital screening algorithms make use of a thresholding mechanism to convert the contone pixel values into halftone micro dot on the recorder grid. The screen function values are arranged as a landscape of "mountains and valleys". The mountains correspond to the high screen function values, the valleys to the low screen function values. The actual shape of this landscape is controlled by the "spot function". Figure 1 demonstrates this with a one dimensional model.

At the time of the screening, the screen function values are compared : dot-by-dot and line-by-line on the recorder grid with the pixel values of the image to be halftoned. If the screen function value is higher than the pixel value at that location on the recorder grid, the laser beam will be modulated "on", and will create a black microdot on the film. Because of the arrangement of the screen function values, SMALL clusters of halftone dots will occur on the film at light parts of the image, while LARGE clusters of microdots will appear at the dark parts of the original image. In this way, the lightness information of the pixels in the original image is transformed into halftone dots with a corresponding size.

As was suggested before, the arrangement of the screen function values controls exactly how the dots grow from small to large for decreasing lightness levels of the pixels. The arrangement also controls the relative position (ruling and angle) of centers of the halftone dots. It turns out that the distinction between most halftoning algorithms can be made in the way that the screen function values are obtained at the time of screening. Some algorithms perform all the mathematical evaluations to obtain the proper screen function value for every element of the recorder grid on the fly; other algorithms do the calculations off line, store the results in a matrix, and just call the prestored values in the appropriate order at the time of the screening itself.

Figure 2 explains how the most elementary form of "simple rational tangent screening" works. The threshold values are precalculated off line for one dot and stored in a square matrix, the size of the matrix determines the size of the dots that it will generate. At the time of the screening, the matrix elements are called in a certain order and used to halftone the contone data. The operation is equivalent to the placement and replication of the matrix elements under the screening angle on the recorder grid.

The frequency and angle of such a screen is given by (see figure 2) :

$$alfa = \arctan (A/B);$$

$$freq = res/sqrt (A*A + B*B);$$

res: resolution.

Since every dot in this method is obtained from the replication of one and the same matrix on the recorder grid, it is a requirement for this method to work that the four corners of the matrix be coincident with the points of the recorder grid. This explains why the method is called "rational tangent", because the arctangent of the angles it produces is always equal to the ratio of the two integer numbers A and B. This has serious implications with respect to the number of rulings and angles that can be generated with this method, and the accuracy by which the conventional angles and rulings can be approximated.

Figure 3 demonstrates that, especially for small cells (a), the angle of replication cannot be precisely controlled. (The angular accuracy becomes better when the tile is larger). In the field of color printing, it has proven to be difficult to achieve combinations of angles and rulings for the cyan, magenta and black separations that result in Moiré free color reproduction. U.S. Patent Nos. US-A-3,657,472, US-A-4,084,183, US-A-4,443,060 and US-A-4,537,470 all contain suggestions on how to reduce the problems that occur in printing when combinations of these screens are used.

In the "preferred embodiment" of Patent No. 4,084,183 it is explained that, if small congruent parcels for the four screens are used, a repeating micro structure is obtained, that, not unlike the conventional rosette, will not be disturbing to the eye.

An improvement of rational tangent screening is obtained by making use of "supercells" that, instead of containing only one dot, contain only one dot, contain "m" dots horizontally and vertically. In that case, it is only required that the four corner points of the SUPERCELLS fall on the recorder grid, rather than the four corner points of every individual dot. Since the size of "supercell" is larger than an individual dot, a higher precision can be achieved in approximating the conventional angles. The method is demonstrated in Figure 4. It is to be noted that the angles that are obtained are still "rational tangent angles", since their arctangent is still equal to the ratio of two integer numbers A and B.

Frequencies and angles of the screens obtained with this algorithm are given by :

$$alfa = \arctan A/B);$$

$$freq = m*res/sqrt(A*A + B*B);$$

res : resolution.
m : multiplicity.

In order to minimize Moiré in color reproduction, much effort has been spent in the past to develop screening algorithms that are able to approximate the conventional screening angles with very high precision. These algorithms are generally referred to as "irrational tangent screening algorithms", since the goal of these algorithms is to approximate the 15 and 75 degree angles (which have an irrational tangent) with the highest possible precision. Most of these algo-

rithms produce screen function values by sampling one period of the spot function under a given angle and at a certain frequency. This technique is shown in Figure 5. The sampling angle and frequency determine the angle and ruling of the screen that will be obtained. It can be shown that, if the tangent of sampling angle is truly an irrational number (i.e. cannot be expressed as the ratio of two integer numbers), the same xy position in the screen function period will never be sampled twice.

There are two classes of implementations possible for these irrational tangent screening algorithms. In the first implementation, the value of the sample in the screen function period is evaluated on the fly. Since this mathematical evaluation has to be done fast, special hardware is usually necessary to achieve the required speed. The method is also limited to spot functions that do not require too much calculations. U.S. Patent US-A-4,419,690 describes a possible implementation.

Another approach comprises precalculating the screen function values of one period, and storing them in a two dimensional matrix, the size of which is typically 32x32 or 64x64 elements. Sampling the screen function period can be done by picking the precalculated matrix element that lies closest to the xy position of the sample point. However, in order to reduce artifacts that result from periodic rounding off effects in addressing the matrix, noise is usually added to the xy position coordinates of the samples, and special averaging techniques have to be used. These techniques are described in U.S. Patent US-A-4,449,489, US-A-4,456,924, US-A-4,700,235 and US-A-4,918,622. These special averaging techniques make the implementation of irrational tangent screening algorithms relatively complex and expensive.

Having described the Moiré analysis of these existing screening systems, their performance in color printing will be described after a brief introduction on a technique for analysing Moiré in color printing.

The principle of Moiré analysis is an interaction between the frequencies and angles of the rasters in color reproduction which can best be studied by means of vector diagrams in the "frequency domain". Every dot raster can be represented by two orthogonal vectors with their length corresponding to the frequency of the screen, and their angle corresponding to the angle of the raster. The effect of the "harmonic frequencies" of the rasters is omitted.

The interactions between two or more rasters correspond to all possible combinations of sum and differences of the original vectors.

It has been experimentally verified that in color printing, the single most important source of Moiré is associated with the second order Moiré (*) resulting from the superimposition of the cyan, magenta and black rasters. The goal in color reproduction is to obtain a Moiré period from these rasters that is INFINITELY large. Figure 6 shows that this is indeed the case when conventional screening is used. The sum of the 15 degree magenta component and the 135 degree black component coincides exactly with the 75 degree cyan component, resulting in a zero frequency (and hence an infinite period) for the Moiré. Similarly, the sum of the 45 degree black component and the 165 degree cyan component coincides exactly with the 105 degree magenta component. With formulas, the equations are :

$$C\_75 = M\_15 + K\_135;$$

$$M\_105 = K\_45 + C\_165;$$

Figure 7 demonstrates what happens if one of the separations for example the black, is off register. As can be seen, the sum of the 15 degree magenta and 135 degree black does not coincide with the 75 degree cyan vector. The "second order Moiré" does not have a zero frequency in this case, and a periodical pattern will be created, the angle and period of which correspond to the difference vector. It should be understood that as used herein, the term "Second Order Moire" means the Moiré as a result of the interaction between an original component (for example the "cyan"), and a component that is already the result of an interaction between two other original components (for example magenta and black).

The vector diagram in Figure 6 only shows the relation between the lengths and angles of the frequency vectors. In addition to these relations, there is also the relative phase of the three rasters. The relative phase of the three rasters will have an effect on the average overlap between the halftone dots, and will determine what kind of rosette is obtained. Figure 8 (a) shows a Moiré free combination of three rasters (15, 45 and 75 degrees) in a particular phase relation. Figure 8 (b) shows the same set of rasters, but the 45 degree screen is half a period shifted in phase. As can be seen, the "rosette" structure is different in both cases. It is believed that the rosette structure of Figure 8 (a) is the preferable one, because it looks less coarse to the eye when seen from a distance, and because it preserves the gradation better in the shadows. Figure 9 shows a case of Moiré. The relative phase between the sum of the 15 and 75 degree rasters changes continuously, resulting in a shifting rosette. The shortest distance of one complete cycle from one kind of a rosette, to another kind and back to the first kind corresponds to the Moiré period, that would be found from the corresponding vector diagram.

In most practical cases, there is enough symmetry in the generation of a set of screens that it is sufficient to mathematically investigate only one of the two second order Moirés. Figure 6 in that case can be replaced by the simplified diagram in Figure 10, which shows a triangle. If the triangle is "closed", there will be no second order Moiré, if it is open, the second order Moiré can be calculated from the sum of the three vectors. Mathematically this corresponds to :

M_x = F1*cos(alfa1) + F2*cos(90.0-alfa2) - F3*cos(alfa3);

M_y = F1*sin(alfa1) - F2*sin(90.0-alfa2) - F3*sin(alfa3);

M_period = 1.0/sqrt (M_x*M_x + M_y*M_y);

As mentioned before, it is impossible to obtain combinations of the rational tangent screens as they were presented before that cancel out second order Moiré completely. The proof for the most common case, namely a set of three screens that are defined by two integer numbers "A" and "B" follows :

Given : A, B, res.
screen1 : alfa1 : arctan(A/B) ; F1 = res/sqrt(A*A + B*B) ;
screen2 : alfa2 = arctan(1.0) ; F2 = res/sqrt((A-B)*(A-B) + (A-B)*(A-B)) ;
screen3 : alfa3 = arctan(B/A) ; F3 = F1;

An example of such screen set is :
res = 2400 dpi, A = 4, B = 15.
By applying the formulas :

alfa1 = 14.9314 degrees; F1 = 154.5976
alfa2 = 45.0000 degrees; F2 = 154.2778
alfa3 = 75.0686 degrees; F3 = 154.5976

The endpoints of the vectors that correspond to screen1 and screen3 are :

endpoint screen1 :(x0,y0) = (F1*cos(alfa1) , F1*sin(alfa1))
endpoint screen3 :(x1,y1) = (F3*cos(alfa3) , F3*sin(alfa3))

or, because :

F1 = F3, cos (alfa3) = sin (alfa1) , and
sin (alfa3) = cos (alfa1) :
endpoint screen1 :(x0,y0) = (F1*cos(alfa1) , F1*sin(alfa1))
endpoint screen3 : (x1,y1) = (F1*sin(alfa1) ; F1*cos(alfa1))

In order to have no Moiré, the angle of the ideal frequency vector F2' should be 45.0 degrees, and its length should be the distance between the two endpoints of screen1 and screen3.

F2' = sqrt ((x0-x1)*(x0-x1) + (y0-y1)*(y0-y1))
= F1*sqrt (2.0 - 4*cos(alfa1)*sin(alfa1))
= F1*sqrt (2.0 - 4*A*B/(A*A + B*B))
= F1*sqrt((2*A*A + 2*B*B - 4*A*B)/(A*A + B*B))

The real length of the frequency vector screen2 however is :

F2 = res/sqrt ((A-B)*(A-B) + (A-B)*(A-B));
= F1*sqrt(A*A + B*B)/sqrt((A-B)*(A-B) + (A-B)*(A-B));
= F1*sqrt((A*A + B*B)/(2*A*A + 2*B*B - 4*A*B)).

In order to have no second order Moiré, it is required that F2 = F2'.
After working out this equation, the following condition is found

3*z*z*z*z - 16*z*z*z + 22*z*z - 16*z + 3 = 0 with : z = A/B;

This is a fourth degree polynomial in "A/B". The equation has two real roots :

z0 = 2.0 - sqrt (3.0);

z1 = 2.0 + sqrt (3.0);

Both are irrational numbers with arctangents 15.0 and 75.0 degrees, respectively. In other words, no two integer numbers A and B exist that can make the Moiré period infinitely large.

The Moiré period in the above example can be calculated and is 1.56 inches. This will be disturbing in most printing situations.

Moiré analysis of "Supercell" screening results in the following : by using multiple dot "supercells", the angular accuracy of the screens can be made considerably better. The Moiré analysis of the "supercell" approach reveals that, although longer Moiré periods can be obtained, it is also here, for the same reason as in the previous case, impossible to obtain completely Moiré free results. An example is given :

Given :
res = 2400; A = 15; B = 56; m = 3;
screen1 : alfa1 = 14.9951; F1 = 124.1933;
screen2 : alfa2 = 45.0000; F2 = 124.1748;
screen3 : alfa3 = 75.0049; F3 = F1;

The Moiré period in this case is 27.07 inches, considerably better than in the pervious case.

The Moiré analysis in the case of irrational tangent screening is quite simple. If a set of three screens is used that are exactly 30 degrees offset, and that have exactly the same rulings, the frequency diagram will show that the three vectors of the screens form an equilateral, closing triangle, and thus that there will be no Moiré.

EP 0 342 640 A2 discusses a mechanism for not causing the secondary Moiré pattern in rational tangent screening systems. Two screens are defined by the two integer numbers m_0 and n_0, where $m_0<n_0$. The third screen is at 45°, giving three different screening angles :

- $\theta_{15} = \tan^{-1}(m_0/n_0)$ corresponding to 15° :
- $\theta_{75} = \tan^{-1}(n_0/m_0)$ corresponding to 75° ; and,
- $\theta_{45} = 45°$.

The screening systems corresponding to 15° and 75° are defined in a supercell with pre-angled halftone dots, having a tile size of a_0 (integer) microdots. The third screen is oriented at 45° exactly and has a tile size of a_1 (integer) microdots. The following relation is obtained : $a_0 = (n_0-m_0)\cdot a_1$ (21) . Based on the above equation (21), Table I in EP 0 342 640 A2 calculates numbers of screen lines, and thereby discloses screening systems, which obey or do not obey this relation. The relevant entries of that table are set out in Table I below, along with a computed value of $(n_0-m_0)\cdot a_1$ for comparison with a_0.

Table I

Line #	1	2	3	4	5	6	7	8	9	10
a_1	50	51	40	35	27	26	25	22	20	17
m_0	1	1	1	1	5	5	1	4	1	1
n_0	3	4	3	3	18	17	3	15	3	4
a_0	100	153	80	70	349	314	50	241	40	51
$(n_0-m_0)\cdot a_1$	100	153	80	70	351	312	50	242	40	51

In the reference it is recognised that certain combinations have relatively long periods in the Moiré pattern. In fact, according to the above equations for computing M_period, the following Moiré periods, expressed in inches, are obtained for Line # 5, 6 and 8 - corresponding to nominal line numbers of 120 and 150 LPI - as set out in Table II below :

Table II

Line #	5	6	8
M_period	1.4754	1.2783	1.6604

The reference asserts that especially in the lines of 150 and 120, there are no combinations without Moiré pattern

at the scanning pitch of 11.25 μ. Therefore the reference solves this problem by using the scanning pitch of 12.5 and 22.5. This solution requires a variable pitch of the recorder, which may be difficult to achieve. Both from Table 1 and Table 2 in the reference, it can be noticed that a solution for the secondary order Moiré problem is disclosed for θ_{45} = 45° and "m/n" = 1/3 or 1/4 only. This corresponds to two screening systems, i.e. :

- $[\tan^{-1}(1/3), 45°, \tan^{-1}(3/1)] \sim [18.4°, 45°, 71.6°]$; and,
- $[\tan^{-1}(1/4), 45°, \tan^{-1}(4/1)] \sim [14.0°, 45°, 76.0°]$.

These two screening systems have the disadvantage that their "precision" with respect to the conventional screening angles [15°, 45°, 75°] is not within 1°. Moreover screen angles between the pair of screens (θ_{75}, θ_{15}) are not between 29.0 and 31.0 nor between 59.0 and 61.0 degrees different, neither for the (1/3, 3), nor for the (1/4, 4) system i.e. :

$$\theta_{75} - \theta_{15} = \tan^{-1}(3) - \tan^{-1}(1/3) = \tan^{-1}(4/3) = 53.13°$$

$$\theta_{75} - \theta_{15} = \tan^{-1}(4) - \tan^{-1}(1/4) = \tan^{-1}(15/8) = 61.93°$$

Therefore, there is clearly a need for a screening system, free from second order Moiré, that gives more freedom and accuracy in the choice of the screen angles and screen rulings.

OBJECTS OF THE INVENTION

It is accordingly a general object of the invention to provide an improved screening system and method for printing reproduction of images.

It is a specific object of the invention to provide a screening system that eliminates second order Moiré.

It is another object of the invention to provide a screening system that can utilize screen angles that are rational tangent angles.

It is still another object of the invention to provide a screening system that can utilize screen angles that are irrational tangent angles.

It is a further object of the invention to provide a method for producing halftone screen function values utilizing a rectangular "tile" having dots hat are preangled under a rational tangent angle.

SUMMARY OF THE INVENTION

The main features of the present invention are as follows : the screening system for printed reproduction of images comprises three separation screens. The separation screens have different screen angles and each screen has rulings. At least two of the screen rulings are different and at least one vector sum of two frequency components of one pair of screens is equal to at least one of the frequency components of the other screen or of a multiple or submultiple of the at least one frequency component. Screen angles that are offset by thirty degrees or a multiple thereof and in which the vector lengths of the frequency components are identical are excluded.

DETAILED DESCRIPTION OF THE INVENTION

The object and features of the present invention will best be understood from a detailed description of a preferred embodiment thereof selected for purposes of illustration and shown in the accompanying drawings, in which :

Fig. 1	is diagrammatic one dimensional model of prior art electronic screening;
Fig. 2	is a diagram illustrating the principle of prior art rational tangent screening;
Fig. 3	is a diagram illustrating the achievable angular accuracy with prior art small and large cell sizes;
Fig. 4	is a diagram illustrating the prior art rational tangent screening with a "supercell";
Fig. 5	is a diagram illustrating the principle of the prior art irrational tangent screening process;
Fig. 6	is a vector diagram illustrating the interaction of three rasters at 15, 45 and 75 degrees with identical rulings to achieve no Moiré;
Fig. 7	is another vector diagram illustrating the production of Moiré as a result of the interaction of three rasters at 15, 45 and 75 degrees with non-identical rulings;
Fig. 8(a)	is an image created by the superposition of three degradees from 0% to 90% rasterized at 15, 45 and 75 degrees with identical rulings which produces a dot centered rosette;
Fig. 8(b)	depicts the result of having offset the 45 degree raster over half a period which yields a clear centered rosette;

Fig. 9 illustrates the Moiré produced by the interaction of three rasters with angles of 15, 45 and 75 degrees with slightly different rulings;

Fig. 10 is a simplified vector diagram illustrating the interaction of orthogonal screens;

Fig. 11 is a vector diagram illustrating the screening system of the present invention that produces zero second order Moiré;

Fig. 12 is a diagram illustrating the geometry of a cell with preangled dots in accordance with the present invention;

Fig. 13 is a vector diagram for Moiré analysis showing that when the three vectors form a closed triangle, second order Moiré is eliminated;

Fig. 14 is a diagram showing a preangled tile, placed and replicated under a rational tangent angle;

Fig. 15 illustrates the overlay of the "15", "45" and "75" degree screens of the present invention using the same "tile" size; and,

Fig. 16 illustrates the use of the 15, 45 and 75 degree screens having the same tile size to "lock" the screens with respect to each other so that there is no accumulation of relative position error with a concomitant consistent rosette across the printed reproduction.

Having analyzed that the absence of Moiré in conventional screening systems is explained by the fact that in the frequency domain the sum of the frequency components of one pair of screens exactly coincides with a component of the third screen, "second order Moiré" also can be completely cancelled out in the case of non conventional screening systems. Figure 11 shows a case where the rulings of the three screen sets are non-identical, yet, by an appropriate selection of the angles, it is possible to obtain exactly the same conditions as in the conventional technology to cancel out the second order Moiré. If the deviations in angle and ruling from the conventional case are small (equal to or less than 1.0 degrees and preferably equal to or less than 0.5 degrees), screening results can be obtained that are virtually indistinguishable from the results obtained with conventional screening, when it comes to Moiré, rosette structure, printability and overall microscopic and macroscopic appearance.

It is also possible to cancel out second order Moiré in this way, if particular combinations of RATIONAL TANGENT screens are used. This is significant, since other methods that use rational tangent screens are not completely free of second order Moiré as was demonstrated, while the methods that generate "irrational tangent" screens are more complex and more expensive in implementation.

Figure 12 shows the geometry of a cell with multiple preangled dots. The angle of the screen that such a tile generates is determined by the ratio of the two integer numbers A and B. The period of the screen is proportional to the size of the tile. The following relations can be immediately derived from Figure 12 :

alfa : screen angle;
Period : screen period in inches;
TS : tilesize expressed in number of recorder dots.
res : recorder resolution expressed in dots/inch.

$$alfa = \arctan(A/B),$$

$$TS*(1/res) - Period*sqrt(A*A + B*B);$$

The following relations can also be derived from the previous :

freq : line ruling of the screen;
shades : average number of recorder elements per dot,
dots : number of dots that the tile contains.

$$freq = res*sqrt(A*A + B*B)/TS;$$

$$shades = TS*TS/(A*A + B*B);$$

$$dots = A*A + B*B.$$

The tile in Figure 12 can be seen as one screen period that, if replicated horizontally and vertically creates a con-

tiguous screen. It is emphasized that the way of creating a rational tangent screen here is fundamentally different from the method described in Figure 4 : In Figure 4, a cell with multiple dots, angled parallel to the cell boundary, is replicated under a rational tangent angle, while in Figure 12, a cell with pre-angled dots is replicated horizontally and vertically. There is a certain symmetry when both techniques are compared.

It should also be clear that, if the parameters TS, A and B are all multiplied or can be divided by the same integer number, equivalent sets of tiles are obtained which will produce the same angles and rulings.

It is possible to combine screens obtained from "tiles" as above that produce results that are completely Moiré free, and that are indistinguishable from the traditional screening systems. The conditions to obtain such screening systems can be summarized as follows :

1. The tilesizes TS of all the three screens are the same.
2. The first screen is defined by two integer numbers A and B, and has an angle alfa1 = arctan (A/B) ;
3. The second screen is defined by two integer numbers C and D, and has an angle alfa2 = arctan (C/D) ;
4. The third screen is defined by two integer numbers E = (B-D) and F = (C-A), and has an angle alfa3 = arctan ((B-D)/(C-A)) ;
5. The difference between alfa1 and alfa3 is 30 degrees +/- 0.5 degrees.
6. The difference between alfa2 and alfa3 is 30 degrees +/- 0.5 degrees.
7. The rulings of the three screens are the same within +/- 2.0%.
8. If two integer numbers X and Y that define the angle of a tile have a common divisor, the parameters TS, X and Y may be replaced by their original values divided by the common divisor.
9. The integer numbers TS, X, Y that define any of these screens can be multiplied by any integer number, since this results in an equivalent tile.

An example of such a screen system is :

res = 2400 dpi;
screen1 : TS1 = 627; A = 11; B = 33;
screen2 : TS2 = 627; C = 34; D = 7;
screen3 : TS3 = 627; E - 26; F - 23;

This screening system is equivalent to the following screening system :

screen1 : TS1 = 57; A = 1; B = 3;
screen2 : TS2 = 627; C = 34; D = 7;
screen3 : TS3 = 627; E = 26; F = 23;

If the formulas for screening angle and ruling are applied on these screening parameters, the following results are obtained :

screen1 : alfa1 = 18.4349 degrees; freq1 = 133.1485 l/i.
screen2 : alfa2 = 78.3664 degrees; freq2 = 132.8731 l/i.
screen3 : alfa3 = 48.5035 degrees; freq3 = 132.8731 l/i.

Using these values in the formulas to calculate the Moiré period, produces
Moiré_x = 0.0; Moiré_y = 0.0; M_period = infinite.

This means that this combination of screens, although different from the conventional case, and each of them having a rational tangent, is completely Moiré free.

The proof that screen sets in general that meet the requirements stated above are free from "second order Moiré" follows.

Given a set of three screens obtained from the same tilesize TS, and with rational tangent angles defined by the following pairs of integers :

screen1 : A, B;
screen2 : E, F;
screen3 : C, D; with E = B-D; F = C-A;

The angles of the screens that these tiles generate are of course :

screen1 : alfa1 = arctan (A/B) ;

screen2 : alfa2 = arctan (E/F) ;
screen3 : alfa3 = arctan (C/D) ;

And the frequencies are :

screen1 : freq1 = res*sqrt (A*A + B*B)/TS ;
screen2 : freq2 = res*sqrt (E*E + F*F)/TS ;
screen3 : freq3 = res*sqrt (C*C + D*D)/TS ;

Figure 13 shows the vector diagram that corresponds to this screening system, with a rectangular frame around it. The "unit of length" in the drawing is equal to res/TS. Since $B = E + D$ and $C = F + A$, the triangle of the three vectors closes, and the screening system is indeed, despite the fact that it consists of rational tangent screens, free from any "second order Moiré".

Calculation of the parameters specified in Figures 12 and 13 is performed by the following program :

```
# include <math.h>
# include <stdio.h>
# define IMAX 10000
/*** CALCULATE OPTIMAL PARAMETERS***/
/* Parameters : angle "alfa1" and frequency "freq" of the first
screen of a Moiré free set, and angular tolerance "atol" to
approximate the angles. The program returns parameters
"TS,A,B,C,D,E and F" as specified in Figures 12 and 13.*/
calc_par (alfa1, freq, atol, res, TS, A, B, C, D, E, F)
double alfa1, atol, freq, res;
int *TS, *A, *B, *C, *D, *E, *F;
{
int dum1, dum2, n1, n2;
double ftol, PI, alfa3, x;
PI = 2.0*acos(0.0);
ftol = 1.0-2.0*cos(PI/3.0 + atol);
ratio_approx (tan(alfa1), atol, A, B);
alfa3 = alfa1 + PI/3.0;
ratio_approx (1.0/tan(alfa3), atol, D, C);
x = sqrt((double)((*A)*(*A) + (*B)*(*B))/((*C)*(*C) + (*D)*(*D)));
ratio_approx (x, ftol, &n2, &n1);
*A = n1* (*A);
*B = n1* (*B);
*C = n2* (*C);
*D = n2* (*D);
*E = (*B) - (*D);
*F = (*C) - (*A),
*TS = res*sqrt ((double) (*E)*(*E) + (*F)*(*F) ) / freq + 0.5;
}
/***RATIO APPROXIMATION***/
/*Parameters : "x" and "tol". Results : two numbers "i" and "j",
the ratio of which approximates "x" with a tolerance "tol".*/
ratio_approx (x, tol, i, j)
double x, tol; int *i, *j;
{
*j = 1 ;
for ( (*i) = 1 ; (*i) < IMAX ; (*i)++ )
       {
       *j = (int) ( (double) (*i)/x + 0.5 ) ;
       if (fabs( (double) (*i)/(*j) - x ) < tol) return ;
       }
}
```

```
/***MAIN***/
/* main program*/
main()
{
double alfa1, atol, PI, freq, res;
int i, j, A, B, C, D, E, F, TS;

PI = 2.0*acos(0.0);
res = 2400.0;
while (1)
      {
      printf ("Enter angle and frequency of screen : ");
      scanf  ("%lf %lf", &alfa1, &freq) ;
      printf ("angular tolerance:") ;
      scanf  ("%lf", &atol);
      alfa1 = PI*alfa1/180.0;
      atol  = PI*atol /180.0;
      calc_par(alfa1, freq, atol, res, &TS,&A,&B,&C,&D,&E,&F) ;
      printf ("TS : %d; ab :%d %d cd: %d %d ef: %d %d",
               TS,          A, B,      C, D,     E, F) ;
      }
}
```

The actual production of the screens can be performed using the teaching of U.S. Patent US-A-4,700,235.

It was explained previously that in the "supercell" rational tangent screening method, the dots are oriented parallel to the cell boundary, and that it is the cell itself that is placed and replicated across the recorder grid (see Figure 4), while in the present "tile" method, the cell contains preangled dots, and is placed and replicated horizontally and vertically (see Figure 12).

There is no reason why both methods cannot be combined.

The geometry that is obtained in that case is depicted in Figure 14. The angle alfa and frequency freq can be calculated as :

Given : A, B, C, D, res.

alfa = alfa1 - alfa2;

tan(alfa) = tan(alfa1 - alfa2);
= (tan(alfa1)-tan(alfa2))/(1.0+tan(alfa1)*tan(alfa2));
= (A/B-C/D)/(1.0 + AC/BD);
= (AD-BC)/(BD + AC);

thus :

alfa = artan ((AD - BC)/(BD + AC)).

freq = res*sqrt(CC+DD)/TS; and TS*TS = A*A+B*B;

thus:

freq = res*sqrt((CC+DD)/(AA+BB)).

By having two degrees of freedom to control the angle of the reproduced screen, it is possible to obtain many more angles, or to achieve a much better approximation in achieving a specified angle. This is demonstrated with an example :

res = 2400 dpi; A = 160; B = 280; C = 5; D = 19;
alfa = 15.0013 degrees;
freq = 146.2138 l/i.

Although this method has an advantage on its own, in the sense that with relatively small tiles better angular accuracy can be obtained than with the "super cell" approach (because of the availability of two degrees of freedom), it is primarily useful to rotate a set of three tiles that are already Moiré free by themselves.

Indeed, a set of three rational tangent screens that are matched to have no second order Moiré, can be generated, as explained above, from the same tilesize. If they have the same tilesize, all three can be rotated by the same amounts with the method as explained in Figure 14, and therefore, they will keep the same relative angles and their rulings will change by the same constant factor. The result is that, starting from the geometry of one set of matched tiles, a whole family of screening combinations can be generated in small angular increments. This is demonstrated with the following example :

Given : res = 2400 dpi; C1 = 4; D1 = 15, C2 = 11, D2 = 11, C3 = 15, D3 = 4;

set 1 : A = 0, B = 280;

screen1 : angle1 = 14.9314 degrees; freq1 = 133.0644 l/i;

screen2 : angle2 = 45.0000 degrees; freq2 = 133.3401 l/i;

screen3 : angle3 = 75.0686 degrees; freq3 = 133.0644 l/i;

set 2 : A = 1; B = 280; (all angles increased by 0.2046 degrees)

screen1 : angle1 = 15.1360 degrees; freq1 = 133.0635;

screen2 : angle2 = 45.2046 degrees; freq2 = 133.3393;

screen3 : angle3 = 75.2732 degrees; freq3 = 133.0635;

set 3 : A = 2; B = 280, (all angles increased by 0.4092 degrees)

screen1 : angle1 = 15.3407 degrees; freq1 = 133.0610;

screen2 : angle2 = 45.4092 degrees; freq2 = 133.3367;

screen3 : angle3 = 75.4778 degrees; freq3 = 133.0610; and so on... (up to +7.5 degrees or down to -7.5 degrees).

Each of the above screen sets is free from second order Moiré, since the original set (set 1) is free from Moiré, and since all three screens are rotated by exactly the same angle, and since the rulings are all changed by the same constant factor.

Because of the internal 30 degree symmetry of a screen set, there is no need to rotate it by more than +/- 7.5 degrees. A 7.5 degree rotation would correspond to a change of the rulings by a factor of :

cos(7.5) = 0.991 (less than 1% change).

The significance of this procedure is that by starting from a Moiré free set of rational tangent screens, a complete range of rotated screen sets can be obtained in small angular increments, and with nearly the same rulings.

Referring now to Figure 15, there is shown an overlay of the 15, 45 and 75 degree screens having the same "tile size". It will be appreciated that the preangled dot geometry of one of the screens corresponds to that shown in Figure 12. Since the three screens use the same tile size, they remain "locked" with respect to each other as shown in Figure

16. Under the "locked" condition, there is ni accumulation of relative position error and, therefore, the rosette remains consistent across the printed page.

Having described in detail a preferred embodiment of the invention, it will now be apparent to those skilled in the art, that numerous modifications can be made therein without departing from the scope of the invention as defined in the following claims.

Claims

1. A screening system, for printed reproduction of images, free from second order Moiré, comprising a first, a second and a third separation screen, each screen having

   - a screen angle, different from any screen angle of the other two screens :
   - a screen ruling ; and,
   - a frequency vector, having a length corresponding to said screen ruling and an orientation corresponding to said screen angle ;

   wherein :

   - at least two of said screen rulings are different ; and,
   - a vector sum of two frequency vectors of one pair of screens is equal to a third frequency vector of the other screen or to a multiple or submultiple of said third frequency vector.

2. A screening system according to claim 1, wherein said different screen rulings are the same within +/- 2.0%

3. A screening system according to any of claims 1 or 2, wherein the first, second and third separation screens represent either :

   - colors magenta, cyan, and black ; or,
   - a same color ; or,
   - a tritone.

4. A screening system according to any of claims 1 to 3, wherein for said first, second and third separation screens either :

   - all the screen angles are rational tangent angles ; or,
   - a screen angle of a screen is an irrational tangent angle.

5. A screening system according to any of claims 1 to 4, wherein two of the screen rulings of different screens are identical.

6. A screening system according to any of claims 1 to 5, wherein halftone screen function values are produced comprising the steps of :

   - generating a rectangular tile having dots that are preangled under a rational tangent angle; and,
   - sampling said rectangular tile under a different rational tangent angle to produce halftone screen function values.

7. A screening system according to any of claims 1 to 6, wherein said first, second or third screen :

   - is generated by a recorder, having a recorder grid ; and,
   - comprises halftone dots, the four corner points of which do not all coincide with the recorder grid or a grid having a resolution twice the resolution of said recorder grid.

8. A screening system according to any of claims 1 to 7, wherein said first, second and third screen each comprise a rectangular tile with multiple preangled dots, for creation of a contiguous screen by preangled tile replication, each said tile having a tile size TS_i, $i=1,2,3$, characterised in that each tile size TS_i is equal to, or a submultiple of, a largest tile size TS_{MAX}.

9. A screening system according to any of claims 1 to 8, not including screens having screen angles :

$$[\tan^{-1}(1/3), 45°, \tan^{-1}(3/1)] \text{ ; nor,}$$

$[\tan^{-1}(1/4), 45°, \tan^{-1}(4/1)]$.

10. A method for screening an image, without generating second order Moiré, comprising the step of using a screening system according to any of claims 1 to 9.

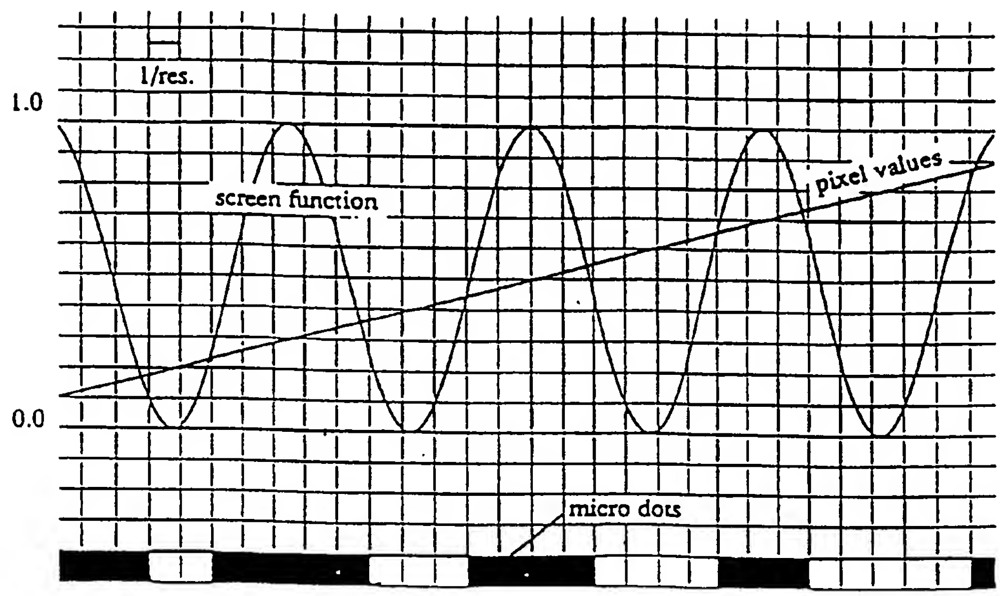

Figure 1: One Dimensional Model of Electronic Screening.

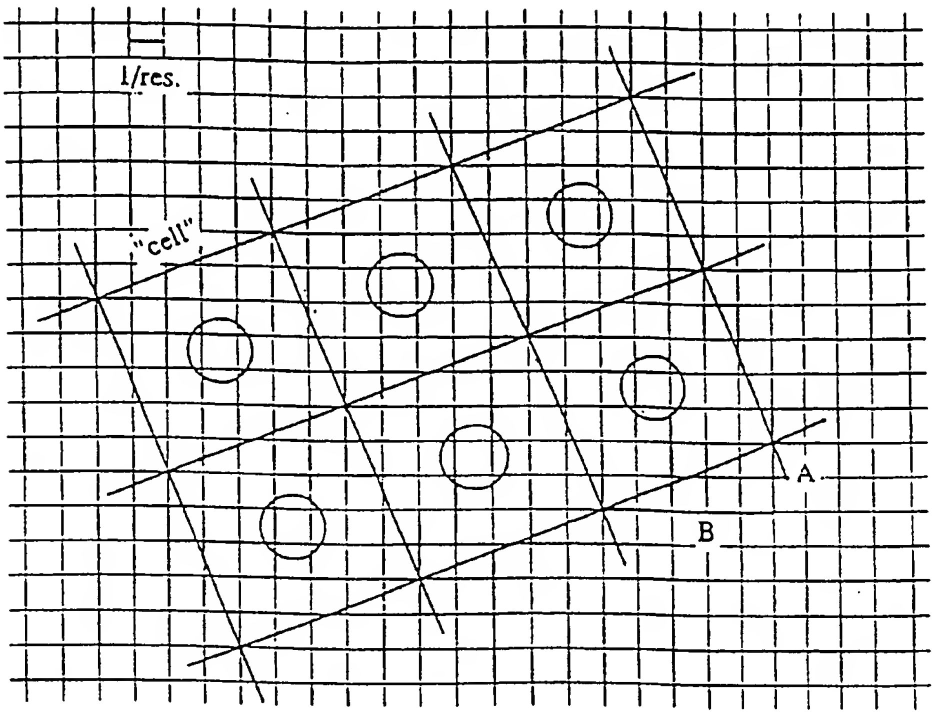

Figure 2: Principle of Rational Tangent Screening.

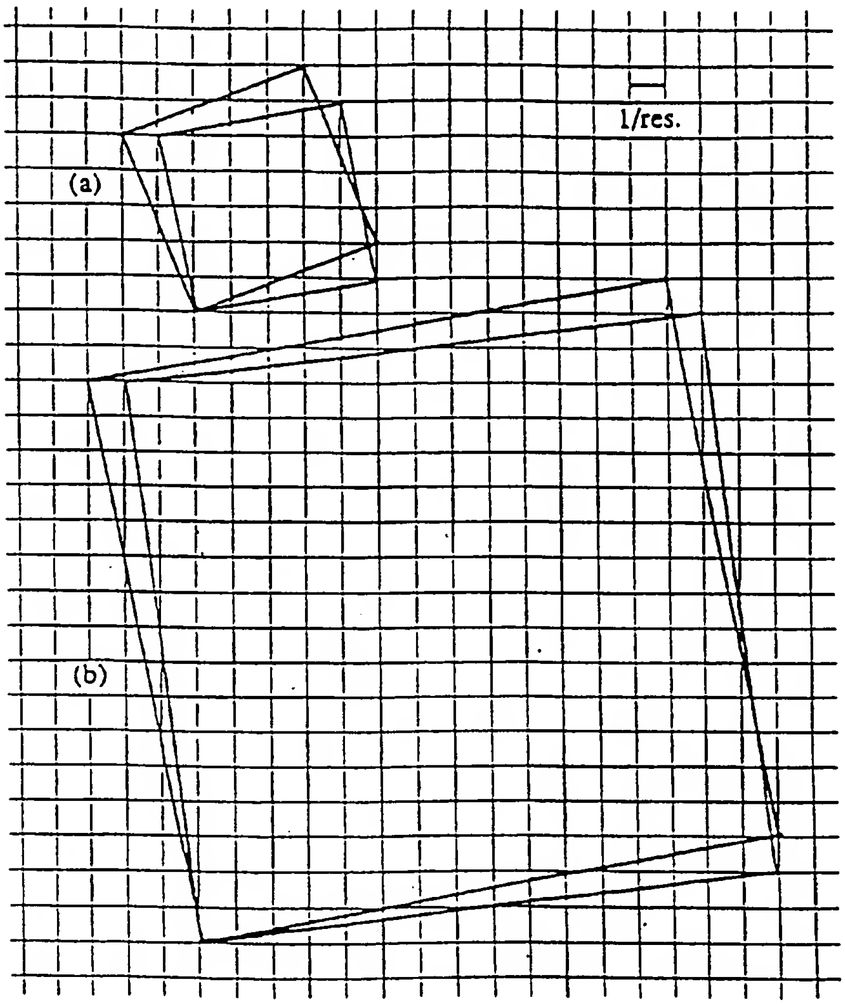

Figure 3: Achievable Angular Accuracy with Small and Large Cells.

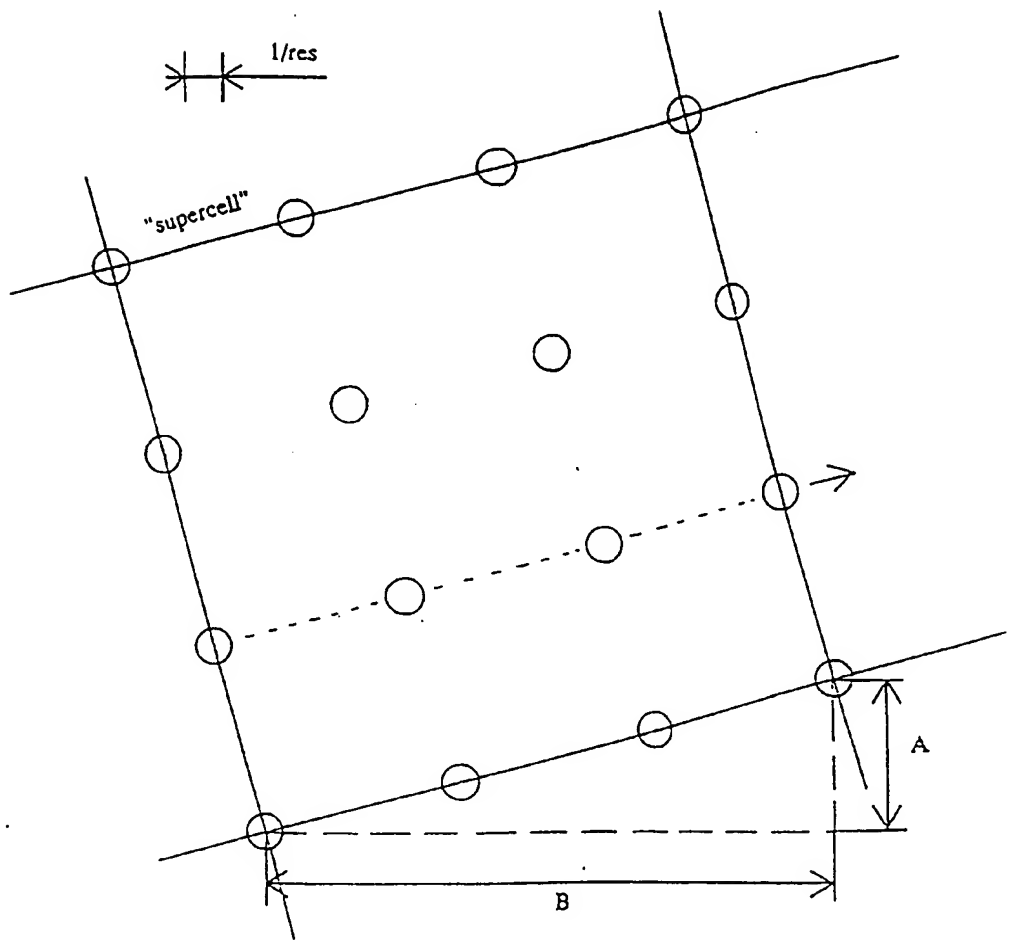

Figure 4: Rational Tangent screening with a "Supercell".

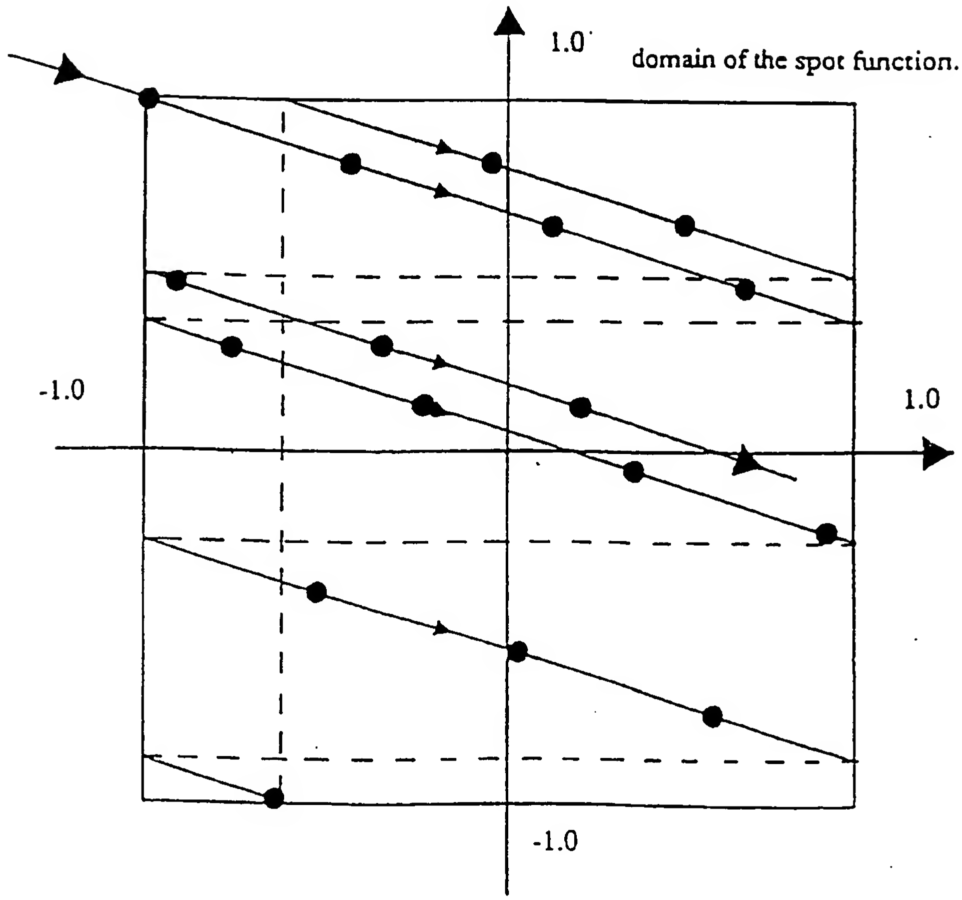

Figure 5: "Irrational Tangent" Screening.

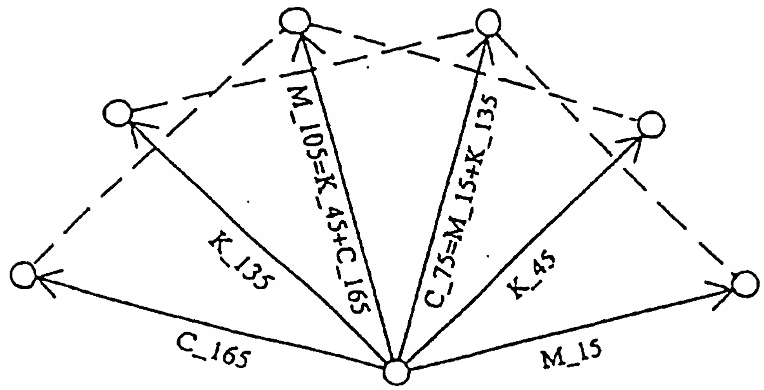

Figure 6: Interaction of three rasters at 15, 45 and 75 degrees, and with identical rulings:

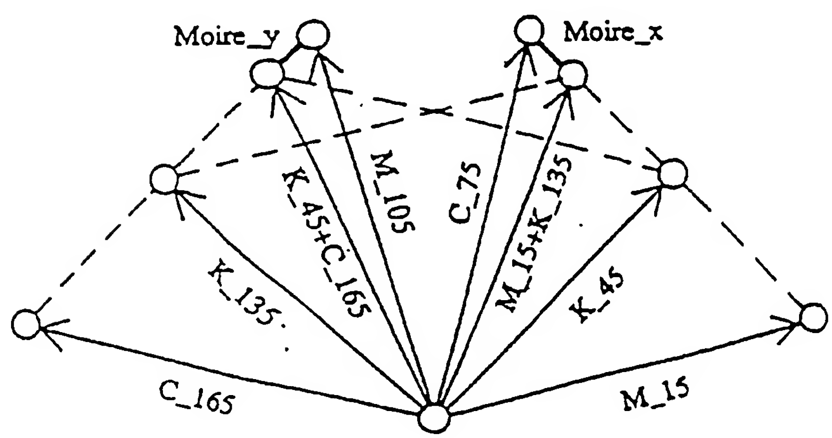

Figure 7: Interaction of three rasters at 15,45 and 75 degrees, and with non identical rulings.

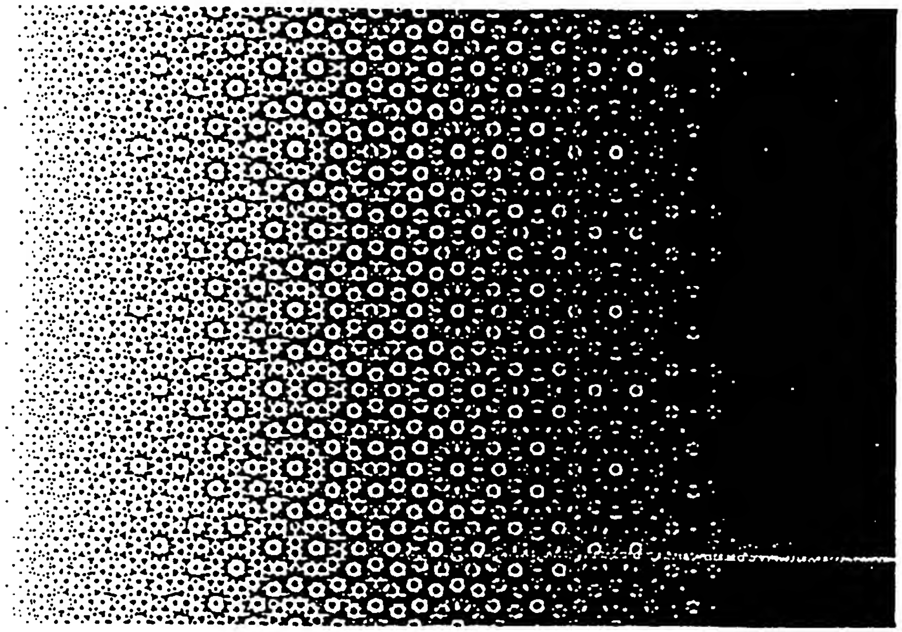

Figure 8 (a): An image created by the superposition of three degradees from 0% to 90%, rastered at 15, 45 and 75 degrees, with identical rulings. The rosette is dot centered.

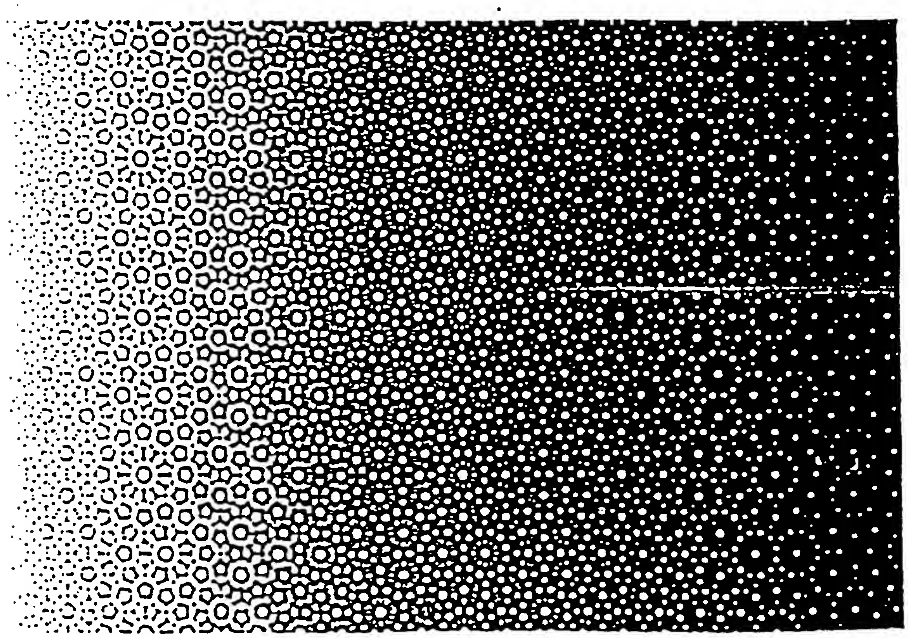

Figure 8 (b): Same as in Figure 2, but by having offset the 45 degree raster over half a period, the rosette is now clear centered.

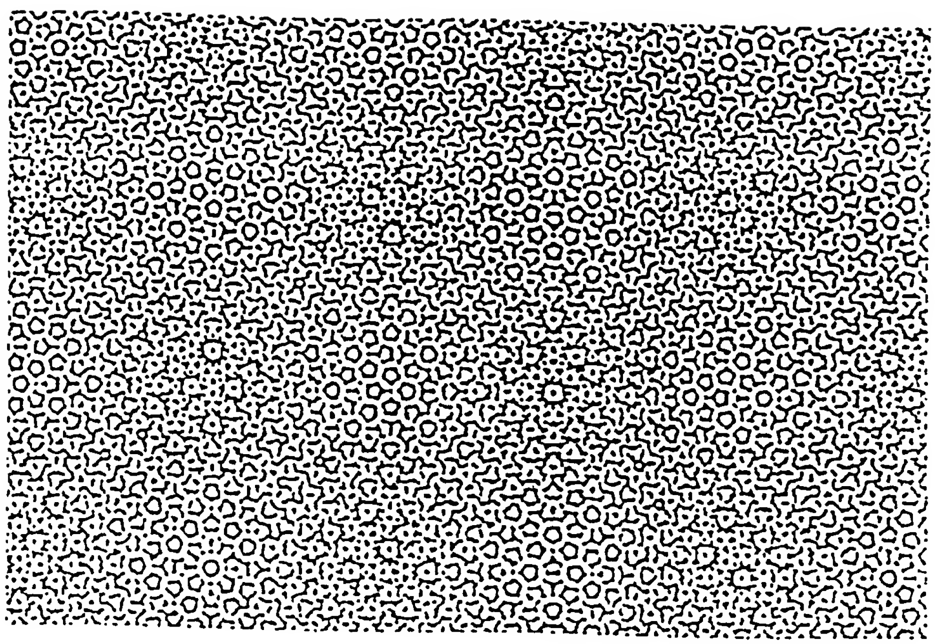

Figure 9: The interaction of three rasters with angles of 15, 45 and 75 degrees and slightly different rulings results in Moire.

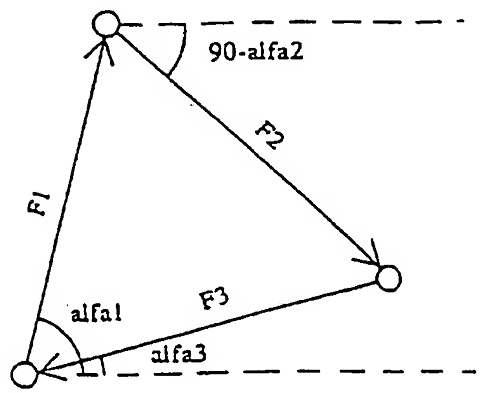

Figure 10: Simplified vector diagram to study the interaction of orthogonal screens.

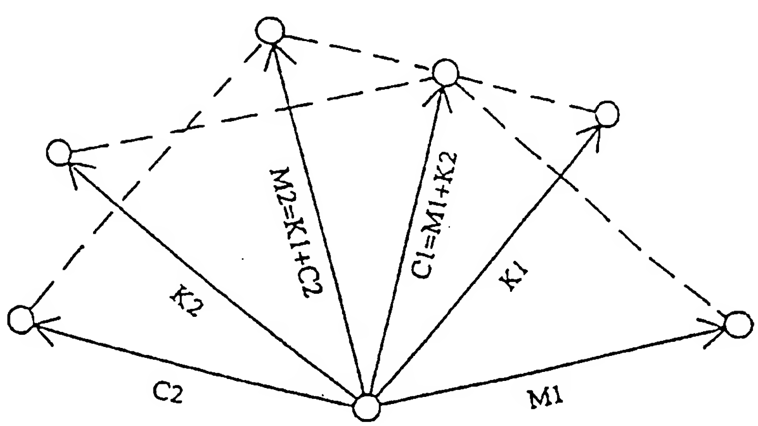

Figure 11: Vector diagram of an unconventional screening system without second order Moire.

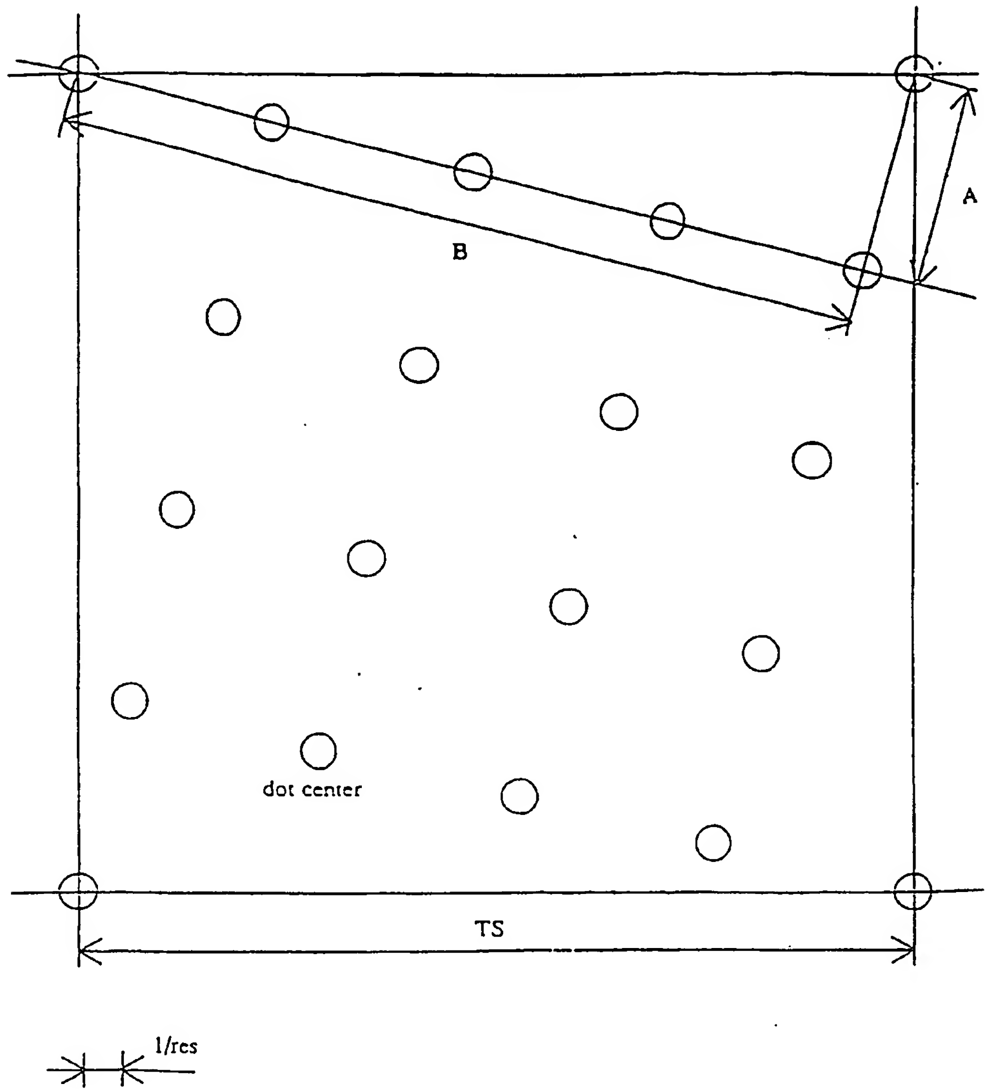

Figure 12: Geometry of a cell with preangled dots.

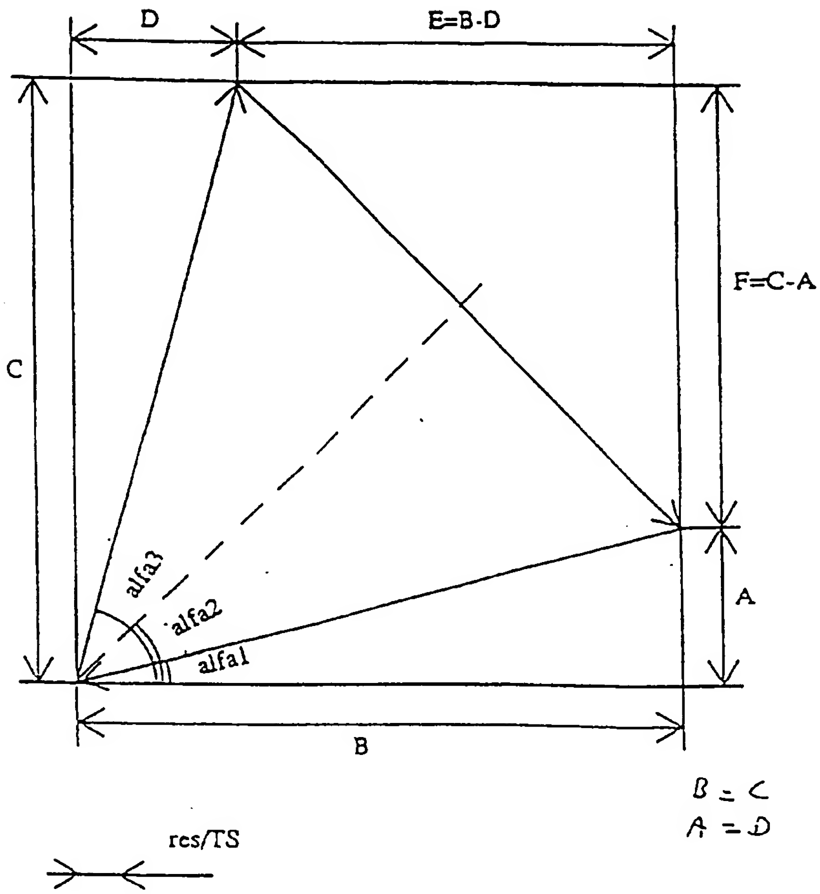

Figure 13: Vector diagram for Moire analysis.

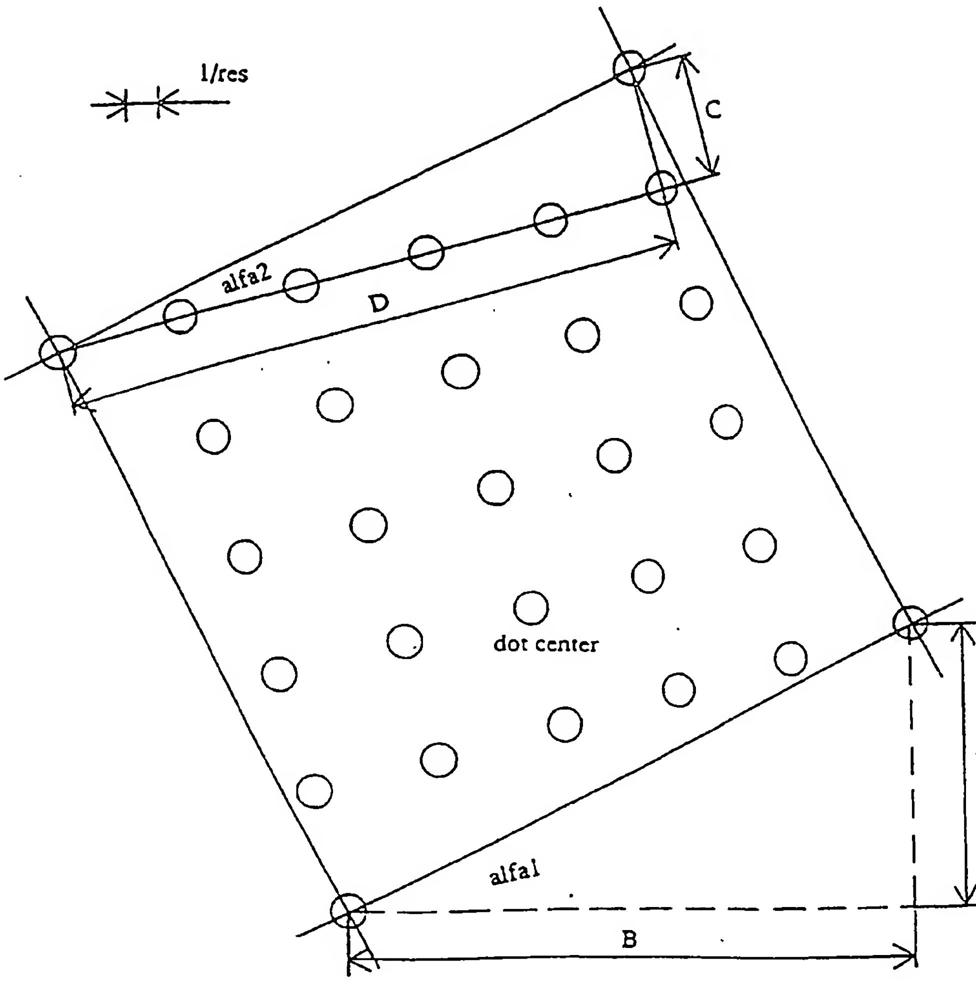

Figure 14: Preangled tile, placed and replicated under a rational tangent angle.

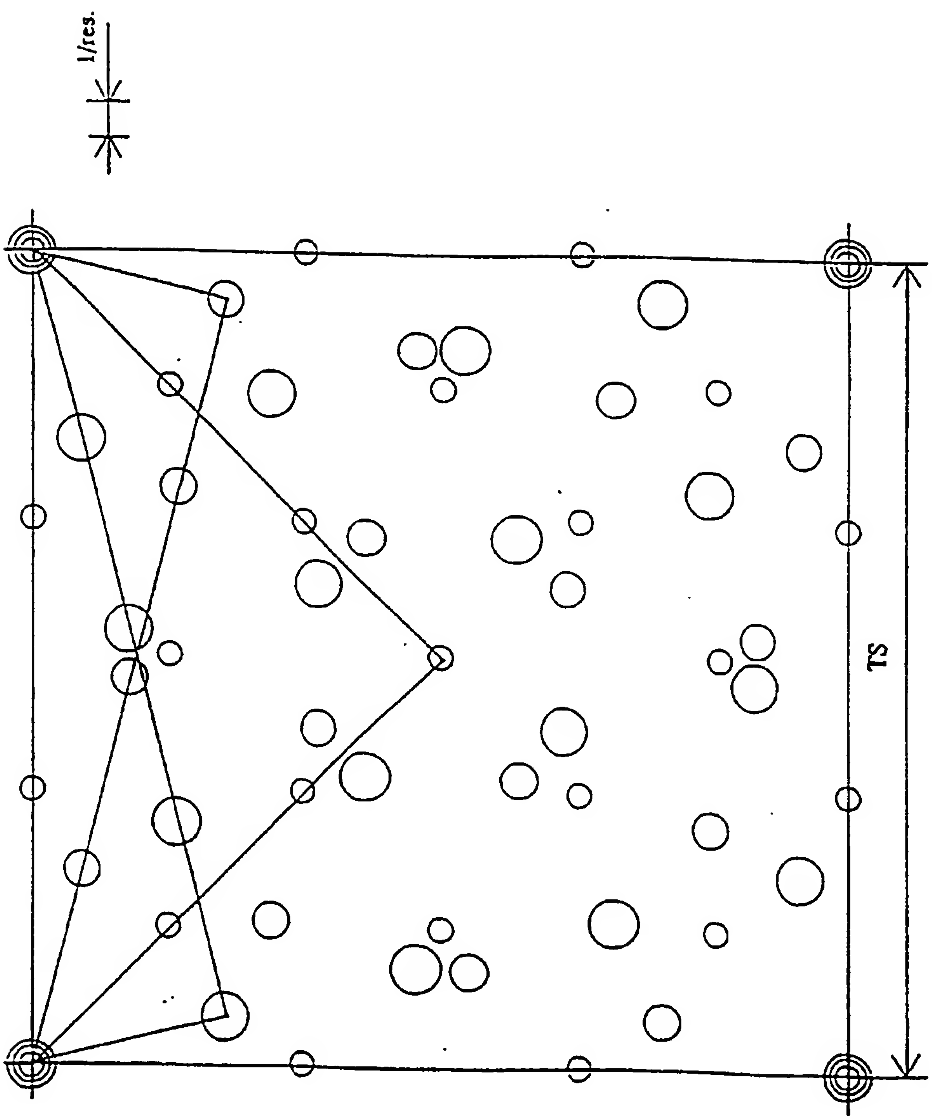

FIG. 15

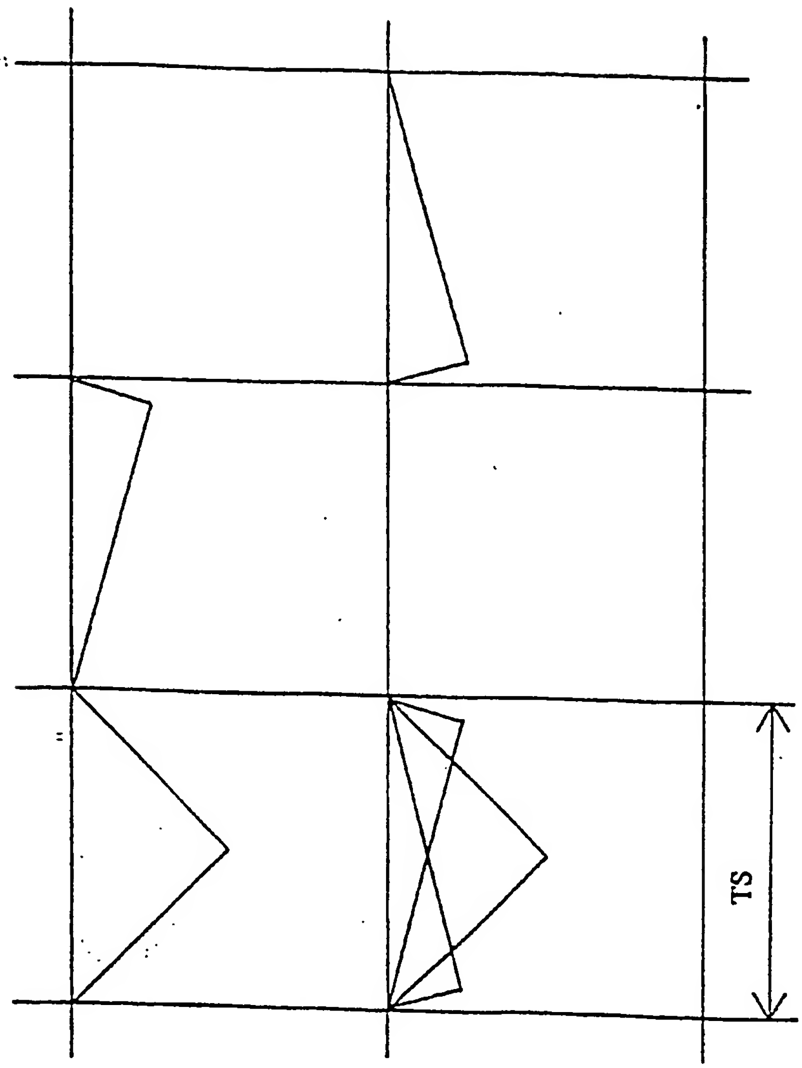

Figure 16